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REVIEWS

Fundamentals of Maxwell's Kinetic Theory of a Simple Monatomic Gas.

By C. TRUESDELL & R. G. MUNCASTER. Academic Press, 1980. 593 pp. $54.00.

As is suggested by the subtitle, viz. 'Treated as a Branch of Rational Mechanics', the authors have written a book on kinetic theory very different from the usual, both in contents and approach. It will appeal to those who, like Truesdell and his school, find the approximations of applied mathematics anathema, especially when these appear to be as insecure as the usual treatments of the kinetic theory of gases make them seem. Unfortunately a consequence of avoiding approximations and 'physical' models is a rather poor yield of practical results, which probably accounts for the absence of any mention of experimental confirmation of the theory. Still, a work that departs substantially from previous approaches is to be welcomed for the new light it may throw on long-standing problems – the historical references alone add considerable interest and for some readers will offset some of the shortcomings later described.

The Prologue clearly states the positivist philosophy of rational mechanics. 'Once the axioms of the theory shall have been stated, we will not alter or "approximate" them. In particular, we shall not mention linearized replacements such as are sometimes claimed valid when this or that quantity is "small", nor shall we consider "models" which replace the theory by another one which is similar to it but mathematically easier.' This approach quickly limits the scope to Maxwellian molecules, i.e. those for which the intermolecular force law is r^{-5}, r being the distance between a pair of interacting molecules. About such special molecules the authors write 'Their prime importance is to promote comprehension of the theory – an importance of which authors of the "physical" kind seem to be insensible... Physicists and chemists will object that those laws of intermolecular force are "unrealistic", they are right but there seems to be little agreement, and little evidence for deciding, what the "true" intermolecular forces are, and anyhow we do not offer this book as a contribution to physics, to chemistry or to computing' – a harsh judgement of the value of their own work unless the 'comprehension' is well achieved.

The book is divided into seven parts, the contents of which will now be briefly described. Most of the material is well-known, although usually presented in a fresh and challenging style; I shall try to identify some of the novel features.

(*a*) *Continuum thermomechanics*

This part is a survey of the ideas of rational mechanics, and includes the principle of 'Material frame-indifference' and the Clausius-Duhem inequality, which are, in fact, approximate rules valid only in the usual *linear* theory of constitutive relations, a type of theory not encouraged in the Prologue. These axioms of continuum mechanics are bogus as general principles, as in fact the authors demonstrate (but curiously do not accept) later in the book [pp. 151, 275, 472]. An exact 'truncation number' Tr is defined on p. 31; on p. 110 it is allowed that this is proportional to the more familiar Knudsen number used as an expansion parameter in the usual account of kinetic theory, so the stage is set for approximations.

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(*b*) *Basic structures of the kinetic theory.*

Among the definitions we find on p. 42 the claim 'The entire purpose of the kinetic theory is to relate the 13 scalar fields [of continuum mechanics] to various circumstances of the gas.' There are of course many other applications of the theory besides calculating transport coefficients. Later the assertion is made that 'it is only the specific, numerical predictions of the kinetic theory that are not mere illustrations of continuum mechanics'. But since kinetic theory fails to confirm two of the important axioms of 'continuum mechanics', at least as defined in part A, this remark cannot be true. Many similar bold assertions throughout the book will give the reader some pause – and hold his interest.

A careful study of summational invariants, and of the collision operator is followed by an account of the Maxwellian distribution function. The authors avoid introducing 'kinetic entropy' as defined via Boltzmann's H-function $h = \overline{\log F}$ – the specific entropy, or 'caloric' as they term it, remains firmly with Clausius. They study 'near Maxwellian' distributions and show that for these h and its flux are proportional to Clausius' entropy η and its flux. Approximations of the kind usually employed are avoided by supplying bounds for the errors.

(*c*) *The Maxwell–Boltzmann equation and its elementary consequences*

Maxwell's equation of transfer and Boltzmann's equation for the distribution function f are explained, and the distinction between kinetic theory and continuum mechanics elucidated. It is shown that Boltzmann's H-theorem does not support the Clausius–Duhem inequality, an important result, but the authors do not find this any reason to reject the latter from continuum mechanics. For the view is held that the continuum and kinetic models are entirely distinct, so that conclusions from one may be at variance with conclusions from the other without concern. Boltzmann's statistical interpretation of his theorem is excluded as not being within the scope of the book – mathematical precision takes priority over explanation. Maxwell's demon is driven back by the holy cross of rational deduction.

(*d*) *Particular molecular models and exact solutions for moments*

Moment equations for Maxwellian molecules are developed; these form the basis of the studies in the rest of the book. Following Maxwell's relaxation theorem, the authors offer some critical views on the use of relaxation times for more general force laws. Yet on p. 197 they introduce the microtime $\tau = \mu/p$, and later exhibit some *error* terms $O(T^n)$, where T is simply the ratio of microscopic to macroscopic time-scales, i.e. a Knudsen number. The quite sound sentence (p. 205), 'We have seen that for small (*sic*) values of T the Stokes–Kirchhoff constitutive relation is borne out fairly (*sic*) well by the kinetic theory' shows how difficult it is to avoid approximations in kinetic theory. Much space is spent on the exact solution for simple shearing of a Maxwell gas; instability is found for sufficiently large T, which provokes interesting comment but no physical explanation. Affine flows are given a similarly careful mathematical treatment.

(*e*) *The system of equations for the moments*

The results obtained in D required no more than third-order moments; in this part complicated expressions are obtained for moments of all orders. The formulae are so tedious that unless computers are permitted, no useful outcome would result. Grad's method of calculating moments is described and his approximations explained and criticised. On p. 275 the fact that kinetic theory is at variance with material frame indifference is noted, but that this *must* have serious consequences for rational continuum mechanics is not deduced (*cf.* similar response in C to the Clausius–Duhem inequality).

(*f*) *Existence, uniqueness and qualitative behaviour*

This contains an account of existence theorems for f in Boltzmann's equation, given initial values of f, almost all of it for Maxwellian molecules.

(*g*) *Grossly and momentally determined solutions and the iterative procedures of the kinetic theory*

Hilbert's formal iterative procedure is discussed at length and gaps in his mathematics pointed out in a patronizing style, although rather more generous than that applied to Chapman and Cowling's contributions to the basic theory of Boltzmann's equation. Following an account of the algebraic basis of the Boltzmann equation, the authors explain the iterative method developed by Enskog and adopted by Chapman, which they later describe as being complicated and obscure. The historical footnotes, while interesting, appear to be written as much to win points as to inform the reader.

The principal contribution to this part is the 'method of stretched fields', which clearly resembles the method of multiple time variable already widely used in treating Boltzmann's equation. The taboo word 'approximation' appears frequently, despite the Prologue, but presumably the authors employ a 'better class' of truncation than those they castigate freely. They apply their expansion procedure directly to the heat flux and stress, rather than to the distribution function, and this, they explain, avoids various problems that arise when Boltzmann's equation is directly involved in the procedure. Their method is therefore closer to Maxwell's method than to Enskog's. They are critical of the 'vague 'method used by Enskog to remove time derivatives, and advance a different method that achieves the same objective. They give general expressions for the Burnett coefficients, and also expressions for the entropy and its flux correct to second-order in their expansion parameter, which proves to be the Knudsen number in disguise. The conclusion is again reached that the theory does not support the Clausius–Duhem inequality of continuum mechanics, as noted above.

The final chapters deal with the 'Maxwellian iteration of Ikenberry & Truesdell', which is a method of repeated differentiation given that the collision integrals are known. It is based on Maxwell's equation of transfer, and when applied to other than Maxwellian molecules, Grad's approximations to the collision integrals are adopted. Surprise is expressed (p. 520) at the appearance of the body force in Burnett's constitutive relations, it being thought that according to the principles of continuum mechanics body forces and contact forces should be quite distinct. But this is merely another manifestation of the fact that constitutive relations *are* frame-dependent.

The above account does not entirely do justice to a remarkable book, for I have selected points that stand out against the methods commonly used in kinetic theory. But the authors have firmly challenged the received view, and must expect weaknesses in their own approach to be identified. The work will certainly be read with interest by those concerned with the foundations of kinetic theory, but I doubt whether its methods will ever replace the Chapman-Enskog approach which is partly based on a physical model, firmly rejected here as not being 'rational'.

L. C. Woods

The Classical Thermodynamics of Deformable Materials. By A. G. McLellan. Cambridge Monographs on Physics, Cambridge University Press 1980. 338 pp. £30.00 (hardback).

In conformity with the aims of the series, this monograph is directed primarily to graduate students and young research workers. Additionally, since the subject is germane to many branches of physics, metallurgy, and geology, the rather elaborate mathematical apparatus is developed *ab initio* and the text is supported by numerous exercises. About one third of the book recounts the basics of matrix algebra, stress analysis, and the geometry of affine deformation. Their treatment is fairly standard but specialists in solid mechanics will find the choice of symbols eccentric (admittedly, however, the author faced a severe problem overall with notation). There is, though, a striking novelty, whose subsequent ramifications confer an idiosyncratic quality on the entire book: the author eschews conventional polar decomposition of a deformation gradient (into a pure strain followed by a rotation) and instead factors out a *triangular* matrix with positive diagonal elements (this matrix itself involves a strain-dependent rotation). His claim that this has 'unique importance in the treatment of thermodynamic stability of solids' deserves detailed comment; this is better left to the end of the review. Regrettably, there is no mention of general measures of strain nor of the associated objective measures of stress generated by work-conjugacy (widely used since 1970); this synoptic viewpoint and unified derivation would have been valuable in appreciating different sets of state variables and in obtaining connexions between the corresponding scalar or tensor state functions. As it is, the author restricts himself to just three types of stress besides Cauchy: (i) nominal (which he calls Boussinesq); (ii) the conjugate of Green's strain; (iii) an unnamed conjugate of the triangular factor mentioned. More generally, to judge by the references, the author seems unfamiliar with current mechanics of solids (his latest citation is Truesdell & Noll 1965).

Moving on to classical thermodynamics, the first and second laws are taken for granted, and likewise the concept of entropy. Of chief concern is the use of Jacobians and Legendre transformations of state functions, more especially when the mechanical variables are chosen in the author's preferred ways. There follow chapters on the corresponding moduli and compliances; coefficients of anisotropic expansion; specific heats; and on the numerous interrelations under arbitrary loading. The subject matter is tedious but the exposition is clear; however, it is noticeable that the densest and least attractive mathematics always occurs when the triangular decomposition is invoked. This central part of the book is rounded off by a concentrated survey of crystal classes, integrity bases, Coxeter groups, and the like; this is emphatically for experts only.

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In the concluding chapters the author makes contact with his own specialisms: stability of variable or multiphase systems (solid or fluid), and the theory of phase transitions and coexistent Dauphiné twins. *Inter alia,* detailed attention is given to the limiting values of thermodynamic quantities at an instability. This authoritative and up-to-date account is no doubt the principal attraction of the book and its *raison d'être.* These particular topics are not within my own competence, so my final observations are focused on the prior discussion of stability of a single elastic phase. It is nowadays well understood that, under load, mechanical stability is a purely relative notion, in that it depends on a specification of the environment and its reaction up to second-order terms in the displacement. While making some gestures in this direction, the author does not appear to be wholeheartedly committed, since he gives prominence to just three environments. These are (i) all-round rigid constraint, (ii) all-round fluid pressure, (iii) dead loading alongside smooth constraints allowing only such homogeneous variations in deformation as may be represented by triangular matrices. Case (i) is impracticable but classical, and leads to Hadamard's necessity criterion (van Hove's converse is not mentioned); case (ii) is practicable and has been satisfactorily treated in the mainstream literature (the author substitutes his triangular decomposition, with no apparent advantage); case (iii) is ostensibly designed as another vehicle for the same decomposition. I did not succeed in following the analysis to its end; but in any event the claim of 'unique importance' must be adjudged nonproven in the general context.

R. Hill

Wind Power Principles: Their Application on the Small Scale. By N. G. Calvert. Charles Griffin & Co. Ltd, London, 1979. 122 pp. £6.20.

On the dust sheet this book is described as 'a well-considered, expertly written work which provides the information the experimenter must have before venturing on any building programme. It is not a highly academic work nor, indeed, is it a catalogue of commercial products and designs'. This aptly describes the level of this little book of 120 pages. It starts with an excellent summary of the history of windmills although, perhaps, there could have been more description of the post- and tower-mills and their relative merits. The next two chapters deal with terminology and some elementary mechanics and fluid mechanics. The author notes that high-speed windmills can be noisy and that in old mills it was common to incline the shaft from the horizontal to avoid interaction between the tower and the sails. There are drawings of Cretan mills, post- and tower-mills, the Tjasker mill, and the American and a modern propeller mill.

The chapters on the aerofoil and the sail cover briefly their essential flow features and characteristics. The chapter on the vertical-axis windmill deals mainly with the Flettner and Savonius types, but mentions only very briefly the 'egg beater' or South Rangi mill.

After a review of the problems and methods of control the author then turns to practical matters, including methods of testing, calculation of power, and strengths and weaknesses of materials. An outline is given of the procedure used by the author for the construction of a pilot plant. The book concludes with a few miscellaneous

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examples of the application of windmills and a discussion of the difficult problem of the storage of the energy generated by them.

The book can be read quickly, and in spite of its elementary approach contains a valuable summary of the factors which experience has shown need to be considered when contemplating the construction of small windmills. It also contains warnings which those who are more ambitious would do well to heed.

W. R. HAWTHORNE

SHORTER NOTICES

An Introduction to Catastrophe Theory. By P. T. SAUNDERS. Cambridge University Press, 1980. 144 pp. £9.50.

Catastrophe theory crops up in numerous contexts, including many fluid mechanical problems, particularly problems of stability and bifurcation. This brief, clear, and attractive presentation of the subject, although making only infrequent reference to fluid mechanical problems, should be both interesting and useful for fluid dynamicists who wish to understand what the theory is all about. The treatment is relaxed, discursive, and requires little in the way of mathematical sophistication, and for the most part, the discussion is limited to the seven elementary catastrophes, the fold, cusp, swallowtail, elliptic umbilic, hyperbolic umbilic, butterfly and parabolic umbilic.

Nonlinear Partial Differential Equations in Engineering and Applied Science. Edited by ROBERT L. STERNBERG, ANTHONY J. KALINOWSKI and JOHN S. PAPADAKIS. (Lecture Notes in Pure and Applied Mathematics Series, vol. 54.) Marcel Dekker Inc., 1980. 504 pp. Sw.Fr. 125.

This volume contains 28 papers from the Conference on Nonlinear Partial Differential Equations in Engineering and Applied Science, sponsored by the Office of Naval Research, and held at the University of Rhode Island in June 1979. The range of topics is wide, covering aspects of numerical analysis and bifurcation theory on the one hand, and applications to topics in fluid mechanics, elasticity, metallurgy and quantum field theory, on the other. In short there is something for everyone. The papers are reproduced by direct photography of the authors' typewritten MSS, and the general standard of preparation of these MSS appears to be high.

Proceedings of 1980 Heat Transfer and Fluid Mechanics Institute. Edited by Melvin Gerstein and P. Roy Choudhury. Stanford University Press, 1980. 251 pp. $22.50.

The theme of the 1980 meeting (the 27th in this well-known series) was the problem of chemical reaction in heat transfer and fluid flow. Only 5 of the 18 papers presented in this volume in fact deal with this topic. The remaining papers treat a wide range of problems in both fluid mechanics and heat transfer, by a variety of numerical and analytical methods. Again the volume has been produced by direct photography of authors' manuscripts and in this case the quality is far from uniform.

The meeting also included a number of invited lectures, but the volume does not include these, nor does it indicate even the titles of these lectures.

Fluidmechanik. Band 1. By E. TRUCKENBRODT. (2nd edn.) Springer, 1980. 371 pp. DM 136.

The first edition of this book, under the title 'Strömungsmechanik', was reviewed in this Journal by N. Rott (vol. 45, 1971, p. 624). The contents of the first edition have been expanded and divided into two volumes for the second edition. This first volume deals with the fundamentals and with constant-density fluids; the second will deal with fluids of variable density together with potential theory and boundary-layer theory. In three chapters, this volume presents the physical properties of fluids, the fundamental equations of fluid mechanics (including some thermodynamics) and the elementary mechanics of hydrostatics, pipe and channel flow.